IJESRT

INTERNATIONAL JOURNAL OF ENGINEERING SCIENCES & RESEARCH TECHNOLOGY

ROLE OF SELECTED MEDICINAL PLANTS IN DIABETICS: AN EVALUATION BY $\left(\frac{G'}{G}\right)$– EXPANSION AND HOMOTOPY ANALYSIS METHODS

Shankar Rao Munjam[1], Sonia Sharma[2], Kondagurley Sandhya Rani[*3], Alok Sharma[4], Xianting Ding[5]

[1] School of Naval Architecture, Ocean and Civil Engineering, Shanghai Jiao Tong University, Shanghai. 200240, China.

[2] Department of Mathematics, Dayawati Modi Public School, Modinagar 201204 Uttar Pradesh, India

[*3] Department of Mathematics, Government Junior College, Rebbena, Komurambheem-Asifabad, 504292 Telangana, India

[4,5] Department of Biomedical Engineering, MED-X Research Institute, Shanghai Jiao Tong University, Shanghai 200030, China

ABSTRACT

Diabetes Mellitus (DM) is a chronic disease characterized by a deficiency in insulin production and its action or both. There are a number of herbal medicines have been recommended for the treatment of Diabetes. The traditional plant medicines were used throughout the world for a range of Diabetic representations.   Recently, Mathematical and Computational techniques have been becoming a united with the Biology and Medicine for consume the computational time. In the present work, three selected medicinal plants i.e. the Helicteres isora Linn. (H. *isora*), the *Portulaca oleracea* L., (P. *oleracea*) and the *Caralluma attenuata* (*C. attenuata*) for Diabetes models of physical significance in two nonlinear differential equations considered and their exact and analytical solutions are obtained. The activity of three plants was confirmed by these techniques and it is found to be safety, efficiency and accuracy in Diabetic. Thus, the obtained results from these techniques are a very good agreement with the laboratory experimental results.

KEYWORDS: Diabetes; Medicinal Plants; Herbal Medicine; Analytical Solution.

I.   INTRODUCTION

Diabetes is chronic pathology which is characterized by hyper glycaemia caused by defective insulin action, insulin secretion, or the combination of both. Diabetes and its complications continue to be a major problem in the world population [1]. Currently, available synthetic drugs for treating this disease are found to be associated with many adverse effects. However, plants and plant-derived products have proven to be effective and safe in the treatment of various types of diabetes mellitus. The use of plants in medicine is an ancient practice in different parts of the world for the preventive and curative purposes [2]. In natural system of medicine, several medicinal plants have found potential use as the blood sugar lowering agents. Experimentally induced diabetes in animals has provided significant depth into the biochemical and physiological derangement of the diabetic state. Many of them have been scientifically explored for their usefulness in managing diabetes [3].

Numerous and varied reports abound in literature on studies conducted to investigate the significant effects of medicinal plants on various pathophysiological complications of diabetes mellitus. In the present study, three following medicinal plants were considered on the basis of their previously published work. Helicteres isora Linn. (H. *isora*) is found to be significant in diabetes. Cumulative research findings on root extract of H. *isora* showed various anti-hyperglycemic and hypolipidemic study signifying the potential use of the extract in the treatment of type-2 diabetes [4-6].

Portulaca oleracea L., (*P. oleracea*) is commonly known as Ma-Chi-Xian in Chines. This plant possesses a wide spectrum of pharmacological properties like antidiabetic, antioxidant, anti-antiulcerogenic and anticancer activities [7]. Its seeds and polysaccharides have been reported for significant antidiabetic potential in streptozotocin (STZ)-induced hyperglycemia and oxidative stress [8, 9]. *Caralluma attenuata* (*C. attenuata*) also known as 'Kundaetikommu' indigenously. This plant is known for its antihyperglycemic activity [10]. Ethnopharmacological claims support for its use in gastric ulcers and diabetes [11].

Mathematical techniques has been united with the art and individuality of biology and medicine, providing indispensable tools for the design [12,13]. Considering this view about the comprehensive mathematical analysis of selected signify their efficacy data in the treatment of Diabetes Mellitus. In our previous study confirmed the above selected medicinal plants for their anti-Diabetic response with help of statistical and mathematical tools [14].

Mathematical Models in Biology and Medicine were new theory to developing the computational effort with numerical techniques [15] as well as approximate analytical methods [16]. The importance of analytical tool for nonlinear problems, namely HAM, is employed to solve ordinary differential equations (7)-(8) with initial conditions (5).

The analytical and numerical techniques has been successfully applied to many nonlinear problems in Bio-Maths such as biochemical reaction model of fractional order [17], Model of Diabetic population [18], Insulin-Glucose feedback and regulatory system of Diabetes [19,20], Diabetes Compartment Modelling [21], detecting Diabtes in the Blood [22], and Fractional modelling dynamics of HIV and CD4+T-cells [23]. The biological resources model Homoclinic orbit equations [24] and predator prey system [25] studied on various special cases of the differential equations. All of these successful applications of the analytical and numerical techniques verify its validity for nonlinear problems in science.

The present study based on $\left(\frac{G'}{G}\right)$–expansion method [26-28] and homotopy analysis method (HAM) [29-30], which are exact and analytical solutions for nonlinear ordinary differential equations. The results obtained ensure that the presented procedure needs less work in comparison with these methods and decreases considerable volume of calculation and a powerful tool for solving large amount of other problems in biology and applied mathematics. Finally the effects of the variation of all the parameters are discussed in detail.

II. MATERIALS AND METHODS

This section dealt with the Diabetes recovery model divides the Diabetes rats $N(t)$ into the three distinct subclasses, such are (i) Diabetes individuals $D(t)$ (ii) Diabetes individuals treated with plant extract $T(t)$ and (iii) have recovered from Diabetes $R(t)$.

The total variable Diabetics population size at time is defined as follows:

$$N(t) = D(t) + T(t) + R(t). \tag{1}$$

We consider the Diabetics rats are recruited into the population at per capita rate parameter A, the natural mortality rate parameter B and probability of Diabetics rates E. The model can be represented by the following nonlinear differential equation as

$$\frac{d^2 D(t)}{dt^2} = A - (B + E)D(t) + D'(t)T(t). \tag{2}$$

Diabetes individuals treated with plant extract T. The population of Diabetics rats individuals are generated at the rate parameter B. This population is decreased by death due to natural mortality parameter G, recovery rats H and which disability cannot be cured parameter E as mathematical model.

$$\frac{d^2 T(t)}{dt^2} = B\,D(t) - (G + H + E)T(t) + T'(t)D(t). \tag{3}$$

The recovered population $R(t)$ is increased by Diabetic rates treated who recovered at the rate G and decreases by the natural mortality at the rate is E as modeled by ordinary differential equation as,

$$\frac{d^2 R(t)}{dt^2} = GT(t) - ER'(t). \tag{4}$$

RESEARCHERID

THOMSON REUTERS

[Rani* *et al.*, 6(11): November, 2017]
IC™ Value: 3.00

ISSN: 2277-9655
Impact Factor: 4.116
CODEN: IJESS7

With the initial conditions as

$$D(0) \geq 0, T(0) \geq 0 \text{ and } R(0) \geq 0, \tag{5}$$

From Eqn. (1), the total population density defined [15] as follows

$$D(t) + T(t) + R(t) = 1$$
$$\Rightarrow R(t) = 1 - D(t) - T(t). \tag{6}$$

Without loss of generality we solve (2)-(5), it is enough to consider (2) and (3) as.

$$D'(t) - D'(t)T(t) + (B + E)D(t) - A = 0, \tag{7}$$

$$T'(t) - T'(t)D(t) + (G + H + E)T(t) - BD(t) = 0. \tag{8}$$

Where the prime (′) denotes the derivative with respective to time t.

The feasible region for the (2) and (3) defined as

$$\Omega = \left\{ \left(D(t), T(t) \right) \epsilon \, \delta^2, D(t) + T(t) \leq 1 \right\}. \tag{9}$$

Since,

$$D(t) = 0 \Rightarrow \frac{dD(t)}{dt} = A > 0,$$

$$\text{and } T(t) = 0 \Rightarrow \frac{dT(t)}{dt} = 0. \tag{10}$$

Therefore, Ω is positively invariant.

Solution of Model by $\left(\frac{G'}{G}\right)$–expansion method

In order to obtain an exact solution of the Eqs. (7-8) with initial conditions (5), by $\left(\frac{G'}{G}\right)$–expansion method [26-28].

Suppose that the solution of (7-8) can be expressed by a polynomial in $\left(\frac{G'}{G}\right)$ as follows

$$D(t) = a_m \left(\frac{G'}{G}\right)^m + \cdots, \text{ where m= 0,1,2,3,…..} \tag{11}$$

$$T(t) = b_n \left(\frac{G'}{G}\right)^n + \cdots, \text{ where n= 0,1,2,3,…..} \tag{12}$$

Here $a_m \neq b_n \neq 0$ and the unwritten part in (11-12) is also a polynomial in $\left(\frac{G'}{G}\right)$, but the degree of which is generally equal to or less than $m - 1$ and $n - 1$, the positive integers m and n can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in (7-8).

Clearly, G = G(t) satisfies the second order linear ODE in the form,

$$G''(t) + \lambda G'(t) + \mu G(t) = 0, \tag{13}$$

where λ, μ are real constants and $G' = \frac{dG(t)}{dt}, G'' = \frac{d^2G(t)}{dt^2}$.

Further (13) can be changed into $\frac{d}{dt}\left(\frac{G'}{G}\right) = -\left(\frac{G'}{G}\right)^2 - \lambda\left(\frac{G'}{G}\right) - \mu. \tag{14}$

By using (11-12) it is easily derived that,

$$\text{D}' = m a_m \left(\frac{G'}{G}\right)^{m+1} + \cdots, \tag{15}$$

$$\text{D}'' = m(m+1) a_m \left(\frac{G'}{G}\right)^{m+2} + \cdots \tag{16}$$

$$\text{T}' = n b_n \left(\frac{G'}{G}\right)^{n+1} + \cdots, \tag{17}$$

$$\text{T}'' = n(n+1) b_n \left(\frac{G'}{G}\right)^{n+2} + \cdots \tag{18}$$

Considering the homogeneous balance between $\text{D}''(t)$ and $\text{D}'(t)\text{T}(t)$ in (7) and that between $\text{T}''(t)$ and $\text{T}'(t)\text{D}(t)$ in (8) based on (15-18), we required that

$$m + 2 = m + 1 + n \implies n = 1$$

$$n + 2 = n + 1 + m \implies m = 1$$

So that $m = 1$ and $n = 1$ in (11-12), we can suppose that the solution of (7-8) is of the form as,

$$\text{D}(t) = a_0 + a_1 \left(\frac{G'}{G}\right) + \cdots, \text{ where } a_1 \neq 0 \tag{19}$$

$$\text{T}(t) = b_0 + b_1 \left(\frac{G'}{G}\right) + \cdots, \text{ where } b_1 \neq 0 \tag{20}$$

By using (11) and (19), it is derived that

$$\text{D}' = -a_1 \left(\frac{G'}{G}\right)^2 - a_1 \lambda \left(\frac{G'}{G}\right) - a_1 \mu \tag{21}$$

$$\text{D}'' = -2a_1 \left(\frac{G'}{G}\right)^3 + 3a_1\lambda \left(\frac{G'}{G}\right)^2 + (2a_1\mu + a_1\mu\lambda + a_1\lambda^2)\left(\frac{G'}{G}\right) \tag{22}$$

$$\text{D}'\text{T} = -a_1 b_1 \left(\frac{G'}{G}\right)^3 - (a_1 b_0 + a_1 b_1 \lambda)\left(\frac{G'}{G}\right)^2 - (a_1 b_0 \lambda + a_1 b_1 \mu)\left(\frac{G'}{G}\right) - a_1 b_0 \mu \tag{23}$$

By using (12) and (20), it is derived that

$$\text{T}' = -b_1 \left(\frac{G'}{G}\right)^2 - b_1 \lambda \left(\frac{G'}{G}\right) - b_1 \mu \tag{24}$$

$$\text{T}'' = -2b_1 \left(\frac{G'}{G}\right)^3 + 3b_1\lambda \left(\frac{G'}{G}\right)^2 + (2b_1\mu + b_1\mu\lambda + b_1\lambda^2)\left(\frac{G'}{G}\right) \tag{25}$$

$$\text{T}'\text{D} = -a_1 b_1 \left(\frac{G'}{G}\right)^3 - (a_0 b_1 + a_1 b_1 \lambda)\left(\frac{G'}{G}\right)^2 - (a_0 b_1 \lambda + a_1 b_1 \mu)\left(\frac{G'}{G}\right) - a_0 b_1 \mu \tag{26}$$

By substituting (19-26) into (7-8) and collecting all terms with the same power of $\left(\frac{G'}{G}\right)$ together, the left-hand side of (7-8) is converted into another polynomial in $\left(\frac{G'}{G}\right)$. Equating each coefficient of this polynomial to zero, yields a set of simultaneous algebraic equations as follows

$$2 + b_1 = 0 \tag{27}$$

$$3\lambda + b_0 + b_1 \lambda = 0 \tag{28}$$

$$2\mu + \mu\lambda + \lambda^2 + b_0 - b_0\lambda + b_1\mu + \text{B} + \text{E} = 0 \tag{29}$$

$$a_1 b_0 \mu + a_0(\text{B} + \text{E}) - \text{A} = 0. \tag{30}$$

On solving the above algebraic equations (27-30) by Maple and obtain for a_0, a_1, b_0 and b_1 as follows:

$$a_0 = 1,$$

$$a_1 = \frac{(B+E)-A}{\mu\lambda},$$

$$b_0 = -\lambda,$$

$$b_1 = -2.$$

Substituting above values in (19-20), we get

$$D(t) = 1 + \frac{(B+E)-A}{\mu\lambda}\left(\frac{G'}{G}\right) + \cdots, \tag{31}$$

$$T(t) = -\lambda - 2\left(\frac{G'}{G}\right) + \cdots, \tag{32}$$

Substituting the general solutions of (13) into (31-32) we obtain three types of solutions of the governing equations (7-8) as follows:

(i)   When $\lambda^2 - 4\mu > 0$

$$D_{1,1}(t) = a_0 + a_1\left[-\frac{\lambda}{2} + \frac{\sqrt{\lambda^2-4\mu}}{2}\left\{\frac{K_1\cosh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\right)t + K_2\sinh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\right)t}{K_1\sinh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\right)t + K_2\cosh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\right)t}\right\}\right]$$

$$= 1 + \left(\frac{(B+E)-A}{\mu\lambda}\right)\left[-\frac{\lambda}{2} + \frac{\sqrt{\lambda^2-4\mu}}{2}\left\{\frac{K_1\cosh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\right)t + K_2\sinh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\right)t}{K_1\sinh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\right)t + K_2\cosh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\right)t}\right\}\right] \tag{33}$$

$$T_{1,1}(t) = b_0 + b_1\left[-\frac{\lambda}{2} + \frac{\sqrt{\lambda^2-4\mu}}{2}\left\{\frac{K_1\cosh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\right)t + K_2\sinh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\right)t}{K_1\sinh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\right)t + K_2\cosh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\right)t}\right\}\right]$$

$$= -\lambda - 2\left[-\frac{\lambda}{2} + \frac{\sqrt{\lambda^2-4\mu}}{2}\left\{\frac{K_1\cosh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\right)t + K_2\sinh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\right)t}{K_1\sinh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\right)t + K_2\cosh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\right)t}\right\}\right]. \tag{34}$$

Where K_1 and K_2 are arbitrary constants.

(ii)   When $\lambda^2 - 4\mu < 0$

$$D_{1,2}(t) = a_0 + a_1\left[-\frac{\lambda}{2} + \frac{\sqrt{4\mu-\lambda^2}}{2}\left\{\frac{K_1\cos\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\right)t - K_2\sin\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\right)t}{K_1\sin\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\right)t + K_2\cos\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\right)t}\right\}\right]$$

$$= 1 + \left(\frac{(B+E)-A}{\mu\lambda}\right)\left[-\frac{\lambda}{2} + \frac{\sqrt{4\mu-\lambda^2}}{2}\left\{\frac{K_1\cos\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\right)t - K_2\sin\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\right)t}{K_1\sin\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\right)t + K_2\cos\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\right)t}\right\}\right] \tag{35}$$

$$T_{1,2}(t) = b_0 + b_1 \left[-\frac{\lambda}{2} + \frac{\sqrt{\lambda^2 - 4\mu}}{2} \left\{ \frac{K_1 \cos\left(\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\right)t - K_2 \sin\left(\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\right)t}{K_1 \sin\left(\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\right)t + K_2 \cos\left(\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\right)t} \right\} \right]$$

$$= -\lambda - 2 \left[-\frac{\lambda}{2} + \frac{\sqrt{\lambda^2 - 4\mu}}{2} \left\{ \frac{K_1 \cos\left(\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\right)t - K_2 \sin\left(\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\right)t}{K_1 \sin\left(\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\right)t + K_2 \cos\left(\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\right)t} \right\} \right] \tag{36}$$

(iii)   When $\lambda^2 - 4\mu = 0$

$$D_{1,3}(t) = a_0 + a_1 \left[-\frac{\lambda}{2} + \frac{K_2}{K_1 + tK_2} \right]$$

$$= 1 + \left(\frac{(B+E)-A}{\mu\lambda} \right) \left[-\frac{\lambda}{2} + \frac{K_2}{K_1 + tK_2} \right] \tag{37}$$

$$T_{1,3}(t) = b_0 + b_1 \left[-\frac{\lambda}{2} + \frac{K_2}{K_1 + tK_2} \right]$$

$$= -\lambda - 2 \left[-\frac{\lambda}{2} + \frac{K_2}{K_1 + tK_2} \right]. \tag{38}$$

Therefore, which are the exact solutions of the governing equations (7-8).

Notice that all solutions (33-38) have been checked with Mathematica by putting them back into the governing equations (7-8). It shows that the $\left(\frac{G'}{G} \right)$–expansion method is more powerful in constructing exact solutions of (7-8).

Solution of Model by HAM

In order to obtain an analytical solution of the non-linear differential equations (7) to (8) satisfying the initial conditions (5), we employ Homotopy Analysis Method (HAM) [29-30]. This method is a recently developed approximate analytical method which is very popular amongst the present researchers. It is a special method in the sense that the pure analytic approach to solution procedure but solvable only using Computer Algebra System such as Mathematica. This method was introduced by Chinese mathematician in the literature only in 1992 by Shi Jun Liao and is described in detail [16]. Hence, the details are not presented here for the sake of brevity. However, the main components involved in applying HAM procedure are (i) selecting suitable initial profiles and satisfying the initial conditions of the problem and (ii) selecting an appropriate auxiliary linear operator so that its solutions are simpler to evaluate analytically.

In the present problem, depending upon the Eq. (5), we choose the initial guesses and auxiliary linear operators as follows

$$D_0(t) = 1 - e^t, \quad T_0(t) = 1 - e^{-t}, \tag{40}$$

$$L_D = D' - D'; \quad L_T = T' - T', \tag{41}$$

$$L_D[C_1 e^t + C_2] = 0 \quad \text{and} \quad L_T[C_3 e^t + C_4] = 0. \tag{42}$$

In which C_i's (i=1,2,3,4) are arbitrary constants. It is to be mentioned here that the choice of initial profiles and linear operators are not unique for a given problem, but the faster convergence of the solution depends on the choice of those.

Zeroth and Higher-order deformation Equations

To obtain the HAM solution for the governing Eqns. (7) - (8), in the standard notations followed in any HAM Analysis, let $\gamma \in [0,1]$ be an embedding parameter and c_D and c_T are the convergence control parameters(CCP). Then the zero[th]-order deformation equation takes the form [30] as,

$$(1-\gamma)L_D\big[\widehat{D}(t,\gamma) - D_0(t)\big] = \gamma c_D \mathcal{N}_D\big[\widehat{D}(t,\gamma), \widehat{T}(t,\gamma)\big], \tag{43}$$

$$(1-\gamma)L_T\big[\widehat{T}(t,\gamma) - T_0(t)\big] = \gamma c_T \mathcal{N}_T\big[\widehat{T}(t,\gamma), \widehat{D}(t,\gamma)\big], \tag{44}$$

where

$$\mathcal{N}_D\big[\widehat{D}(t,\gamma), \widehat{T}(t,\gamma)\big] = \frac{\overline{\partial^2 D}(t,\gamma)}{\partial t^2} + \left(\frac{\partial \widehat{D}(t,\gamma)}{\partial t}\right)\left(\widehat{T}(t,\gamma)\right) + \big(B(\gamma) + E(\gamma)\big)\left(\widehat{D}(t,\gamma)\right) - A(\gamma), \tag{45}$$

$$\mathcal{N}_T\big[\widehat{T}(\eta,\gamma), \widehat{D}(\eta,\gamma)\big] = \frac{\overline{\partial^2 T}(t,\gamma)}{\partial t^2} + \left(\frac{\partial \widehat{T}(t,\gamma)}{\partial t}\right)\left(\widehat{D}(t,\gamma)\right) + \big(G(\gamma) + H(\gamma) + E(\gamma)\big)\left(\widehat{T}(t,\gamma)\right)$$
$$- B(\gamma)\left(\widehat{D}(t,\gamma)\right). \tag{46}$$

For l[th]-order deformations equation, we first differentiate Eqs. (43)-(44) l-times with respect to γ; dividing them by $l!$ and then set $\gamma = 0$. Following this procedure we have

$$L_D[D_l(t) - \Omega_l D_{l-1}(t)] = c_D \mathcal{R}_D\left(\widehat{T}(\eta,\gamma), \widehat{D}(\eta,\gamma)\right), \tag{47}$$

$$L_T[T_l(t) - \Omega_l T_{l-1}(t)] = c_T \mathcal{R}_T\left(\widehat{T}(\eta,\gamma), \widehat{D}(\eta,\gamma)\right). \tag{48}$$

Where $\mathcal{R}_D\left(\widehat{T}(\eta,\gamma), \widehat{D}(\eta,\gamma)\right)$ and $\mathcal{R}_T\left(\widehat{T}(\eta,\gamma), \widehat{D}(\eta,\gamma)\right)$ are remainders term of the linear operators such as,

$$\mathcal{R}_D\left(\widehat{T}(\eta,\gamma), \widehat{D}(\eta,\gamma)\right) = \mathrm{D}'_{l-1}(t) + \sum_{j=0}^{l-1}\big[\mathrm{D}'_j(t)T_{l-1-j}(t)\big] + \big(B(\gamma) + E(\gamma)\big)D_{l-1}(t)$$
$$- A(\gamma) \tag{49}$$

$$\mathcal{R}_T\left(\widehat{T}(\eta,\gamma), \widehat{D}(\eta,\gamma)\right) = \mathrm{T}'_{l-1}(t) + \sum_{j=0}^{l-1}\big[\mathrm{T}'_j(t)D_{l-1-j}(t)\big] - \mathrm{B}(\gamma)D_{l-1}(t) + \big(G(\gamma) + H(\gamma) +$$
$$E(\gamma)\big)T_{l-1}(t). \tag{50}$$

Where Ω_ℓ is defined as $\Omega_\ell = \begin{cases} 0, & l \leq 1 \\ 1, & l > 1. \end{cases}$ \hfill (51)

Expanding $\widehat{D}(t,\gamma)$ and $\widehat{T}(t,\gamma)$ in Taylor series with respect to γ,

$$\widehat{D}(t,\gamma) = D_0(t) + \sum_{l=1}^{\infty}[D_l(t)\gamma^l], \tag{52}$$

where, $D_l(t) = \frac{1}{l!}\frac{\partial^l \widehat{D}(t;\gamma)}{\partial \gamma^l}\big|_{\gamma=0}$,

$$\widehat{T}(t,\gamma) = T_0(t) + \sum_{l=1}^{\infty}[T_l(t)\gamma^l], \tag{53}$$

where, $T_l(t) = \frac{1}{l!}\frac{\partial^l \widehat{T}(t;\gamma)}{\partial \gamma^l}\big|_{\gamma=0}$.

For $\gamma = 0$ and $\gamma = 1$ in Eqs. (47)-(48) one may be write as,

$$\widehat{D}(t,0) = D_0(\eta), \qquad \widehat{D}(t,1) = D(t), \tag{54}$$

$$\widehat{T}(t,0) = T_0(\eta), \qquad \widehat{T}(t,1) = T(t). \tag{55}$$

Thus as γ increases from 0 to 1 and $\widehat{D}(t,\gamma)$ and $\widehat{T}(t,\gamma)$ varies from the initial guess, the functions $D_0(t)$ and $T_0(t)$ approaches to the solution $D(t)$ and $T(t)$ of the governing equations respectively. Here, the auxiliary parameters are suitably chosen so that the series solution converges for $\gamma = 1$.

$$D(t) = D_0(t) + \sum_{l=1}^{\infty}[D_l(t)], \tag{56}$$

$$T(t) = T_0(t) + \sum_{l=1}^{\infty}[T_l(t)]. \tag{57}$$

Therefore we get the general approximate analytical solutions D_l and T_l in terms of series solutions evaluated up to l^{th}-order D_l^* and T_l^* along with the solution of the linear operator chosen in the problem as

$$D_l(t) = D_l^*(t) + \mathbf{C}_1 e^t + \mathbf{C}_2, \tag{58}$$

$$T_l(t) = T_l^*(t) + \mathbf{C}_3 e^t + \mathbf{C}_4. \tag{59}$$

We solve the Eqns. (58) - (59) for various values of 'l' starting from $1, 2, 3, ...$ by means of the symbolic computation software Mathematica 10.0.

Computational Procedure

A Mathematica code is written to solve the Eqns. (58) to (59). Starting from known functions for $D_0(t)$ and $T_0(t)$, the next set of functions $D_1(t)$ and $T_1(t)$ are computed using the (58) to (59). Using these two sets, the next set of functions $D_2(t)$ and $T_2(t)$ are algebraically computed and so on. At any stage, the functions obtained are a polynomial of degree l in 'c_D' and 'c_T' are the convergent control parameters, where the coefficients of the polynomial may contain some or all the parameters present in the problem. The computations are carried out up to $l = 50$, to as certain that the values of $D_l(t)$ and $T_l(t)$ obtained at that stage are consistent and accurate up to 7 digits. It is to be mentioned here that the analytical expressions obtained for $D_l^*(t)$ and $T_l^*(t)$ are too lengthy even for the value of $l = 5$. Hence the actual expressions are reproduced here.

Convergence of the solutions

As it is done traditionally in homotopy analysis, the auxiliary parameters 'c_D' and 'c_T' are optimized by drawing these curves with respect to c_D and c_T to find the convergent interval for D(t) and T(t). From these curves it is found that the admissible ranges of c_D and c_T are $-2.3 < c_D < -0.3$ and $-1.5 < c_T < -0.5$. The c_D and c_T -curves are shown in Figure 1. It is clear from Figure. 1 that the numerical values for D(t) and T(t) are insensitive to the value of c_D and c_T chosen from the above mentioned range.

The results are presented in the form of tables and figures in the following sections. In all of the figures, we use $\mathbf{B = 150}$, $\mathbf{E = 13}$, $\mathbf{G = A = 1}$, $-c_D = 0.741625$ and $-c_T = 0.627879$ unless it is mentioned otherwise.

Table 1. The laboratory experimental values of D(t) at t=90.

Treatment	Laboratory experimental values of D(t)
Normal	196.3826822
Control	411.1954257
100mg/kg	737.0110911
250mg/kg	114.3107323
Tolbutamide	516.2874261

Table 2. The $\left(\frac{G'}{G}\right)$–expansion method* and HAM** solutions for D(t) at t=90 when ($A = 1, E = 13,$ $-c_D = 0.600425$ and $-c_T = 0.627879.$)

The population of Diabetics rats individuals are generated at the rate parameter B	*Results of D(90)	**Approximate analytical values of D(90)
120	196.07	196.05756209
140	251.80	253.80752713
160	342.96	340.80804529
180	428.63	427.05817803
220	517.14	516.30751646

Table 3. The laboratory experimental values of T(t) at t=90.

Treatment	Laboratory experimental values of T(t)
Normal	124.188922
Control	560.079518
100mg/kg	196.728036
250mg/kg	329.935951
Tolbutamide	432.806407

Table 4. The $\left(\frac{G'}{G}\right)$–expansion method* and HAM** solutions for T(t) at t=90 when ($B = 150, E = 13, G = A = 1, -c_D = 0.600425$ and $-c_T = 0.627879.$)

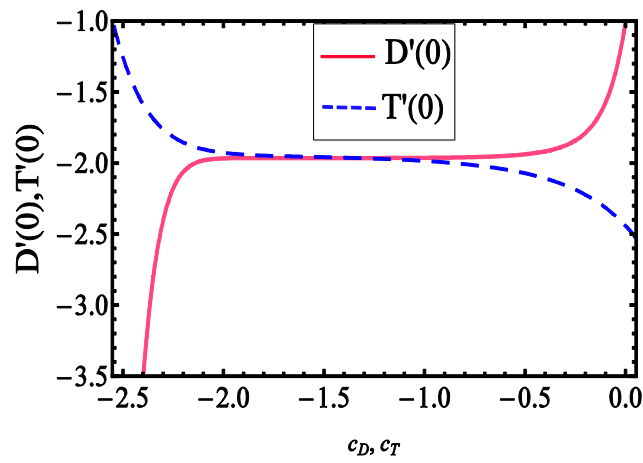| Recovery rats parameter H	*Values of T(90)	*Approximate analytical values of T(90)
10	122.06	124.06286201
15	184.24	183.28266933
20	265.85	265.07552765
25	331.17	330.10820800
30	432.93	432.80355279

Fig. 1 c_D and c_T-curve for the functions $D'(t)$ and $T'(t)$ at t = 0.
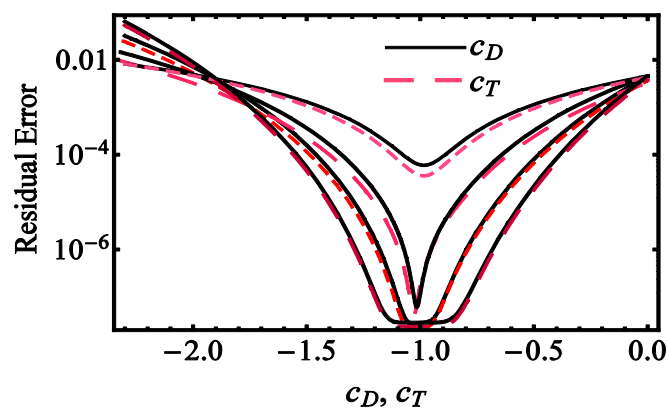

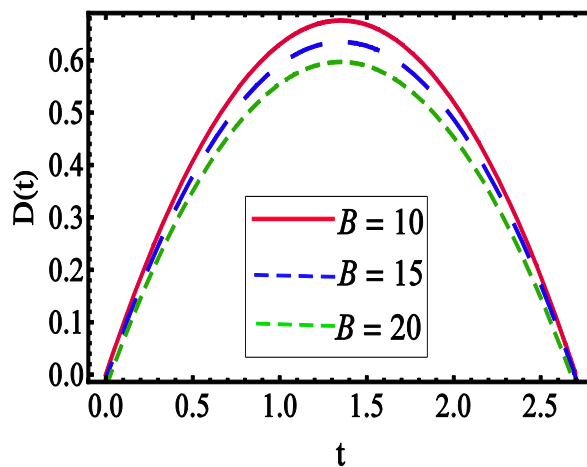**Fig. 2** Residual Error analysis curves.

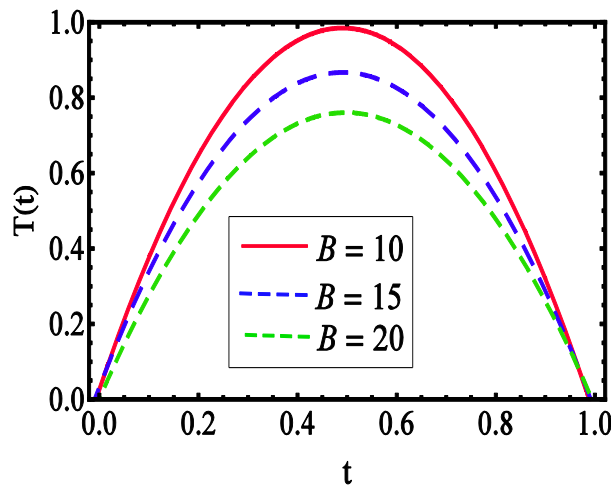**Fig. 3** $D(t)$-curve for the time t at $c_D = -0.741625$ and $-c_T = 0.627879$.

Fig. 4 $T(t)$-curve for the time t when $c_D = -0.741625$ and $c_T = -0.627879$.

Residual Error Analysis

In order to choose a proper value of CCP, the squared residual error is defined

$$\mathbf{E}_D \approx \frac{1}{\ell} \sum_{j=0}^{i} \left[N\left(\sum_{i=0}^{\ell} D_i(j\Delta x) \right) \right]^2, \text{ here } \Delta x = 10/\ell \text{ and } \ell = 25. \tag{60}$$

$$\mathbf{E}_T \approx \frac{1}{\ell} \sum_{j=0}^{i} \left[N\left(\sum_{i=0}^{\ell} T_i(j\Delta x) \right) \right]^2, \text{ here } \Delta x = 10/\ell \text{ and } \ell = 30. \tag{61}$$

From Eqs. (60) and (61) to find c_D and c_T values and obtained these values are used for the entire computations of solution. The HAM based **Mathematica package BVPh 2.0** has been utilized to compute the average residual errors of governing equations. Comparison between c_D and c_T error analysis, it shows very accurate results have seen in Fig.2. The corresponding residual error decreases very rapidly. It is found to be safety, efficiency and accuracy in Diabetic.

III.   RESULTS AND DISCUSSION

The analytical solutions obtained are a set of polynomials in higher powers of c_D and c_T (the Homotopy parameters) whose coefficients contain A, B, E, G, H and E are mentioned above. The solution can be obtained up to any order depending upon the required accuracy. It is often the case that the expressions obtained from HAM are too lengthy even for a polynomial of degree 5 in c_D and c_T. In the present analysis, the computations have been performed up to 25th degree to ascertain the consistency in values, but the results presented in figures are calculated from the 20th degree polynomial in c_D and c_T of the series solution containing all the parameters.

In order to assess the accuracy of present $\left(\frac{G'}{G} \right)$–expansion method and HAM solutions for D(t) and T(t), compared with laboratory experimental values shown in Tables 1-4. It is seen from these tables that, the calculative values of Homotopy analysis at 1% level of significance is less than the laboratory experimental values and hence there is no change in Blood Glucose and Blood Glucose Tolerance. The oral glucose to tolerance test (OGTT) measures the body's ability to use a type of sugar, called glucose that is the body's main source of energy. The extract (Dose 150mg/kg and Dose 300mg/kg) prevented the increase in the Blood glucose levels significantly after glucose load in three selected medicinal plants. Tables 2, 3 illustrates that, the population of Diabetics rats individuals are generated at the rate parameter B and Recovery rats parameter H are increases as values of D(t) and T(t) at t=90 increase. It is also observed that the maximum glucose tolerance is H. *isora* extract at the 90th mins after the glucose loading followed by P. *oleracea* and C. *attenuate*. Only one set of representative result for $D(t)$ and $T(t)$- curves for a various value for the natural mortality rate parameter B are presented in Figures. 3-4. It is observed that, $D(t)$ and $T(t)$-curves increases as natural mortality rate

parameter B increases due to probability of Diabetics rates E.   The probability of Diabetics rates E is fixed at13.0.

IV.   CONCLUSION

The different extract of H. *isora*, P. *oleracea* and C. *attenuata* were tested for anti-Diabetic activity, earlier by glucose tolerance test and this hypothesis conformed by $\left(\frac{G'}{G}\right)$–expansion method and Homotopy Analysis Method (HAM). However, present work will enhance the possible mechanism of action of the different extract for their pharmacological response with help of these two techniques and it confirmed the evidence which was previously scientific validate on animal anti-Diabetics. This novelty will enhance their claim to be used regular anti-Diabetics.

V.   ACKNOWLEDGEMENTS

VI.   REFERENCES

[1] Chakrabarti R, Vikramadithyan RK, Mullangi R, Sharma VM, Jagadheshan H, Rao YN, Rajagopalan, R. Antidiabetic and hypolipidemic activity of Helicteres isora in animal models. Journal of ethnopharmacology 2002; 81(3): 343-349.
[2] Kumar G, Murugesan AG. Influence of Helicteres isora bark extracts on plasma and tissue glycoprotein components in streptozotocin diabetic rats. Journal of Clinical and Diagnostic Research 2007; 4: 330-338.
[3] Raja AB, Elanchezhiyan C, Sethupathy S. Antihyperlipidemic activity of Helicteres isora fruit extract on streptozotocin induced diabetic male Wistar rats. Europ Rev Med Pharm Sci. 2010; 14: 191-6.
[4] Venkatesh S, Reddy BM, Reddy GD, Mullangi R, Lakshman M. Antihyperglycemic and hypolipidemic effects of Helicteres isora roots in alloxan-induced diabetic rats: a possible mechanism of action. Journal of natural medicines 2010; 64(3): 295-304.
[5] Kumar V, Sharma M, Lemos M, Shriram V. Efficacy of Helicteres isora L. against free radicals, lipid peroxidation, protein oxidation and DNA damage. Journal of Pharmacy Research 2013; 6(6): 620-625.
[6] Bhavsar SK, Föller M, Gu S, Vir S, Shah MB, Bhutani KK, Lang, F. Involvement of the PI3K/AKT pathway in the hypoglycemic effects of saponins from Helicteres sera. Journal of ethnopharmacology 2009; 126(3): 386-396.
[7] Alok S, Vijayakumar M, Raj VA, Md T, Unnikrishnan MK. In *vitro* α-Amylase Inhibition and Antihy perglycemic Activity of Helicteres isora in Streptozotocin-Induced Rats. International Journal of Pharmaceutical and Clinical Research 2009; 1(1): 15-18.
[8] Zhou YX, Xin HL, Rahman K, Wang SJ, Peng C, Zhang H. Portulaca oleracea L.: A Review of Phytochemistry and Pharmacological Effects: BioMed Research International (2015). doi: 10.1155/2015/925631.
[9] Sharma A, Vijayakumar M, Rao CV, Unnikrishnan MK, Reddy GD. Action of Portulaca oleracea against streptozotocin-induced oxidative stress in experimental diabetic rats. Journal of Complementary and Integrative Medicine 2009; 6(1): 1–10.
[10] Sharma A, Kaithwas G, Vijayakumar M, Unnikrishnan MK, Rao CV. Antihyperglycemic and antioxidant potential of polysaccharide fraction from portulaca oleracea seeds against streptozotocin- induced diabetes in rats. Journal of food biochemistry 2012; 36(3): 378–382.
[11] Kumar P, Sharma A, Varshney P, Rao CV. Antidiabetogenic and antioxidant effects of Caralluma attenuate extract on streptozotocin induced diabetes in rats. Journal of Pharmacy Research 2013; 7(3): 257-262.
[12] Garg S, Srivastava S, Singh K, Sharma A, Garg K. Ulcer healing potential of ethanolic extract of Caralluma attenuata on experimental diabetic rats. Ancient science of life 2016; 35(4): 222–226.
[13] Srivastava M, Abbas M, Mandal SK. Statistical Evaluation of Biological Activity of Plant Based Products: Anti-diabetic Screening. Statistical Advances in Biosciences and Bioinformatics 2006; 35.

[14] Sonia, S, Rachit, K, Vasista H, Deepankar S. Evaluation of select medicinal plants to assess their evidence as anti-diabetic by using mathematical and statistical techniques. British Biomedical Bulletin 2014; 2 (4):

622-628.

[15] Ullah R, Zaman G, Islam S. Stability analysis of a general SIR epidemic model. VFAST Transactions on Mathematics 2013; 1(1): 16-20.

[16] Liao S. Beyond Perturbation: Introduction to the Homotopy Analysis Method, CRC Press, Boca Raton: Chapman and Hall; 2003.

[17] Zurigat M, Momani S, Alawneh A. The multistage homotopy analysis method: Application to a biochemical reaction model of fractional order. International Journal of Computer Mathematics 2014; 91(5): 1030-1040.

[18] Sandhya, DK, Pandit P. An Ordinary Differential Equation Model of Diabetic Population in New Delhi. Indian Journal of Mathematics and Mathematical Sciences 2011; 1(7) : 45-50.

[19] Chalishajar DD, Stanford CAC. Mathematical Analysis of Insulin-Glucose Feedback System of Diabeties. International Journal of Engineering 2014; 5(2): 36-58.

[20] Makroglou A, Li J, Kuang Y. Mathematical models and software tools for the glucose-insulin regulatory system and diabetes: an overview. Applied Numerical Mathematics 2006; 56(3-4): 559-573.

[21] Chiarella C, Shannon AG. An example of Diabetes Compartment Modelling. Mathematical Modelling 1986; 7(9-12): 1239-1244.

[22] Kwach B, Ongati O, Simwa R. Mathematical Model for Detecting Diabetes in the Blood. Applied Mathematical Sciences 2011; 5(6): 279-286.

[23] Arafa AAM, Rida SZ, Khalil M. Fractional modelling dynamics of HIV and CD4+T-cells during primary infection. Nonlinear Biomedical Physics 2012; 6(1): 1-7.

[24] Zhu C, Long B. The periodic solution bifurcated from homoclinic orbit for coupled ordinary differential equations. Mathematical Method in the Applied Sciences 2016; 40(8): 2834-2846.

[25] Luo D, Wang D. On almost periodicity of delayed predator–prey model with mutual interference and discontinuous harvesting policies. Mathematical Methods in the Applied Sciences 2016; 39(15): 4311-4333.

[26] Kutluay S, Esen A, Tasbozan O. The $\left(\frac{G'}{G}\right)$–expansion method for some nonlinear evolution equations. Applied Mathematics and Computation. 2010; 217(1): 384–391.

[27] Bekir A. Application of the $\left(\frac{G'}{G}\right)$–expansion method for nonlinear evolution equation. Physics Letters A 2008; 372(19): 3400–3406.

[28] Aslan I. Discrete exact solutions to some nonlinear differential-difference equations via the $\left(\frac{G'}{G}\right)$–expansion method. Applied Mathematics and Computation 2009; 215(8): 3140-3147.

[29] Liao S. Homotopy Analysis Method in Nonlinear Differential Equatuions. Springer: New York; 2012.

[30] Seshadri R, Munjam SR. The study of heat and mass transfer in a viscoelastic fluid due to a continuous stretching surface using homotopy analysis method. Journal of Applied Analysis and Computation 2014; 4: 389-403.

CITE AN ARTICLE